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ESD-TDR-62-211

Contractor's Report Number U-1758

INITIAL ORBIT DETERMINATION FROM INDEPENDENT OBSERVATIONS OF SATELLITE POSITION

by

James E. Evans

TECHNICAL DOCUMENTARY REPORT NO. ESD TDR-62-211
1 July 1962

Aeronutronic
A Division of Ford Motor Company
Newport Beach, California

Prepared under Contract No. AF19(604)-7375 for
496L Systems Program Office
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L. G. Hanscom Field, Bedford, Mass.

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ABSTRACT

A computer program has been developed to compute initial satellite orbits from two or more independent observations of satellite position. The technique combines other observations with the initial elements by means of a differential correction scheme to obtain a corrected initial orbit. Extensive experimentation indicates that appropriate selection of observations to be used in the initial computations will yield accurate initial orbits.

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SECTION 1

INTRODUCTION

The Initial Orbit Radar Fix Program with Differential Correction (IORF) is coded for the Philco 2000 computer and is designed to be a part of the Semi-Automatic Programming System at the SPADATS Center in Colorado Springs, Colorado. It functions in conjunction with the Executive Program and receives its input from the Schedule and SEAI Tapes.

The purpose of the program is to compute a set of initial elements, for a satellite orbit, from two or three position fixes. These position fixes may be isolated radar strikes which come from different stations or occur during different revolutions. The method used, is to make a first approximation of the semi-latus rectum, p, and iterate until the computed time between the two observations is within some specified tolerance of the observed time between the two observations (p-iteration).

The option to differentially correct the initial orbit is available. The differential correction technique used is the same as that used in the SGPDC Program. The inclination may be corrected, but the drag coefficient is never corrected. Regardless of whether or not the option to correct the orbit is chosen, the seven standard element cards are always punched.

SECTION 2

FORMULATION

2.1 COMPUTATION OF r FROM A POSITION FIX

Given the quantities:

Observation day number. (in days since 1950.0);

Minutes from Greenwich midnight of observation day to observation time;

Observed azimuth, A, and elevation, h; or topocentric right ascension, α , and declination, δ ;	obtained by ØBSGET routine
Observed slant range, ρ;	

Latitude, ϕ , and east longitude, λ_E, of the observing station;

$-(C+H) \cos \phi$ and $Z = -(S+H) \sin \phi$ for the observing station;

The Greenwich sidereal time at the start of the observation year, θgr_o;	obtained by TLC routine

The following procedure is used to compute the geocentric position vector, $\underline{r}$:

a. Compute the geocentric station vector, $\underline{R}$.

$$\theta gr\ (deg) = \theta gr_o + (\dot{\theta} - 360^o)\ D + \dot{\theta} F$$

Where θgr is Greenwich sidereal time at observation time, $\dot{\theta}$ is the rotation rate of the earth in deg/solar day, and D and F are the observation time in days and fractions of a day, respectively, since the start of the observation year.

$\theta = \theta_{gr} + \lambda_E$ where θ is the observation sidereal time.

$$\underline{R} \quad \begin{aligned} X &= [-(C+H)\cos\phi]\cos\theta \\ Y &= [-(C+H)\cos\phi]\sin\theta \\ Z &= -(S+H)\sin\phi \end{aligned}$$

b. Compute the topocentric position vector, $\underline{\rho}$

$$\underline{L} \quad \left.\begin{aligned} L_x &= \cos\theta\ (\cos\phi\ \sin h - \cos A\cos h\sin\phi) - \sin\theta\cos h\sin A \\ L_y &= \sin\theta\ (\cos\phi\ \sin h - \cos A\cos h\sin\phi) + \cos\theta\cos h\sin A \\ L_z &= \sin\phi\ \sin h + \cos A\cos h\cos\phi \end{aligned}\right\}\ \text{If A, h are observed}$$

$$\underline{L} \quad \left.\begin{aligned} L_x &= \cos\delta\cos\alpha \\ L_y &= \cos\delta\sin\alpha \\ L_z &= \sin\delta \end{aligned}\right\}\ \text{if } \alpha,\ \delta \text{ are observed}$$

$\underline{\rho} = \rho\,\underline{L}$

3) Compute $\underline{r}$ and r

$\underline{r} = \underline{\rho} - \underline{R}$

$$r = \sqrt{\underline{r} \cdot \underline{r}} = \sqrt{x^2 + y^2 + z^2}$$

2.2 PRELIMINARY COMPUTATIONS, INDEPENDENT OF p

Given two position fixes, $\underline{r}_1$, and $\underline{r}_2$, the following computations are made. These calculations are independent of the selection of the semi-latus rectum, p.

a. <u>Compute the Orientation Vectors and Angles.</u>

$$\underline{U}_i = \frac{\underline{r}_i}{r_i} \qquad i = 1, 2$$

$$C = \cos(v_2 - v_1) = \underline{U}_1 \cdot \underline{U}_2$$

$$S = \sin(v_2 - v_1) = \pm\sqrt{1 - C^2} \qquad \text{(both signs are tried)}$$

$$\underline{W} = \frac{\underline{U}_1 \times \underline{U}_2}{S} \qquad (W_z = \cos i)$$

$$\underline{V}_i = \underline{W} \times \underline{U}_i \qquad i = 1, 2$$

$$\sin i = \sqrt{1 - W_z^2}$$

$$i = \tan^{-1} \frac{\sin i}{\cos i} \qquad 0 \leq i \leq \pi$$

$$\Omega = \tan^{-1} \frac{W_x}{-W_y} \qquad 0 \leq \Omega < 2\pi$$

b <u>Calculate Range of p's and Maximum Number of Revolutions Possible</u>

An expression, which describes the curve on which the solution for the semi-latus rectum must fall, can now be derived using the following expressions;

$$e \cos v_i = p/r_i - 1 \qquad i = 1, 2$$

$$e \sin v_1 = \frac{C\ e \cos v_1 - e \cos v_2}{S}$$

$$e \sin v_2 = \frac{e \cos v_1 - C\ e \cos v_2}{S}$$

These equations may be substituted into the expression

$$e^2 = (e \sin v_1)^2 + (e \cos v_2)^2$$

to yield

$$e^2 = a_1 p^2 + a_2 p + a_3$$

where

$$a_1 = \frac{1}{r_1^2} + \frac{1}{S^2} \left(\frac{1}{r_2} - \frac{C}{r_1} \right)^2$$

$$a_2 = 2 \left[- \frac{1}{r_1} + \frac{1}{S^2} (C-1) \left(\frac{1}{r_2} - \frac{C}{r_1} \right) \right]$$

and

$$a_3 = 1 + \frac{1}{S^2} (C-1)^2$$

The equation for e^2 describes a parabola on an e^2, p, plot. To determine the limits of p, the quadratic may be solved for the arbitrary limiting values of $e^2 = 0.95$, ($e = 1$ gives singularity points.)

$$p = \frac{- a_2 \pm \sqrt{a_2^2 - 4a_1 (a_3 - 0.95)}}{2a_1}$$

Therefore, for any $e^2 \leq 0.95$ the roots of the above equations must bound the actual value of p, i.e.,

$$p_{1min} \leq p \leq p_{1max}$$

where

$$p_{1max} = \frac{-a_2 + \sqrt{a_2^2 - 4a_1 (a_3 - 0.95)}}{2a_1}$$

$$p_{1min} = \frac{-a_2 - \sqrt{a_2^2 - 4a_1 (a_3 - 0.95)}}{2a_1}$$

The subscript 1 is used to differentiate these p's from the ones which follow.

Similarly:

$$p_{2min} \leq p \leq p_{2max}$$

where p_{2min} is derived from Kepler's second law

$$p \geq \left(\frac{r_1 \; r_2 \; S}{k_e(t_2-t_1)}\right)^2 = p_{2min}$$

and

$$p = r(1 + e \cos v) \leq 2(\text{the smaller of } r_1 \text{ or } r_2) = p_{2max}$$

There is also the additional requirement that p be always greater than unity. Thus the maximum allowable p, max p, is taken as the smallest value of p_{1max} and p_{2max}. Similarly the minimum allowable p, min p, is taken as the largest value of 1, p_{1min}, and p_{2min}.

The maximum number of revolutions possible during the time interval (t_2-t_1) can be calculated by the following equation:

$$\max N \leq \frac{t_2 - t_1}{P_{co}}$$

where P_{co} is the period of a surface circular satellite.

2.3 p-ITERATION

The interval for p (min p < p < max p) is divided into m parts, where m is a constant and is part of the program input. These divisions may be called Δ p, i.e.

$$\Delta p = \frac{\max p - \min p}{m}$$

There is now a family of p's; min p, min p + Δ p, min p + 2 Δp,...., max p. The p-iteration is now commenced by calculating a function, f_N (min p) for all the integer values of N less than max N. (The derivation of $f_N(p)$ will be shown below) This same procedure is then performed for p min + Δ p, etc.. g_1. If there is a change in the sign of $f_N(p)$ for successive values of p at some N, say between p_1 and p_2 for N*, a root is indicated.

$f_{N^*}\left(\frac{p_1 + p_2}{2}\right)$ is computed and its sign examined to determine on which side of $\frac{p_1 - p_2}{2}$ the root is. If the root is less than $\frac{p_1 + p_2}{2}$, then $f_{N^*}\left(\frac{3p_1 + p_2}{4}\right)$ is computed; if the root is greater than $\frac{p_1 + p_2}{2}$, then $f_{N^*}\left(\frac{p_1 + 3p_2}{4}\right)$ is computed. This halving process is continued until a p is found for which $|f_N(p)| \leq \epsilon_3$, (ϵ_3 is an input to the program).

Given p and N, $f_N(p)$ is computed as follows:

a. <u>Compute the Shape and Size of the Orbit and the Period</u>:

$$e \cos v_i = \frac{p}{r_i} - 1 \quad i = 1,2$$

$$e \sin v_1 = \frac{C\ e \cos v_1 - e \cos v_2}{S}$$

$$e \sin v_2 = \frac{e \cos v_1 - C\ e \cos v_2}{S}$$

where $C = \cos (v_2 - v_1) = \underline{U}_1 \cdot \underline{U}_2$

$$e^2 = (e \cos v_1)^2 + (e \sin v_1)^2 = (e \cos v_2)^2 + (e \sin v_2)^2$$

$$a = p/(1 - e^2)$$

$$n = k_e/a^{3/2}$$

$$P = \frac{2\pi}{n}$$

b. Compute the Mean Longitude, L_i, $i = 1, 2$:

$$\left.\begin{aligned}(1 + \cos i) \sin \ell_i &= U_{yi} - V_{xi} \\ (1 + \cos i) \cos \ell_i &= U_{xi} + V_{yi}\end{aligned}\right\} \quad \text{if } W_z > 0$$

$$\left.\begin{aligned}(1 + \cos i) \sin \ell_i &= - U_{yi} - V_{xi} \\ (1 + \cos i) \cos \ell_i &= U_{xi} - V_{yi}\end{aligned}\right\} \quad \text{if } W_z < 0$$

$$\ell_i = \tan^{-1} \left(\frac{\sin \ell_i}{\cos \ell_i}\right)\left(\frac{1 + \cos i}{1 + \cos i}\right)$$

$$\frac{p}{r_i} \sin (E - v)_i = - \frac{(e \sin v_i)(e \cos v_i)}{1 + \sqrt{1 - e^2}} + e \sin v_i$$

$$\frac{p}{r_i} \cos (E - v)_i = 1 - \frac{(e \sin v_i)^2}{1 + \sqrt{1 - e^2}} + e \cos v_i$$

$$(E - v)_i = \tan^{-1} \left[\frac{(p/r_i) \sin (E - v_i)}{(p/r_i) \cos (E - v_i)}\right]$$

$$e \sin E_i = \frac{r_i}{p} \sqrt{1 - e^2} (e \sin v_i)$$

$$L_i = \ell_i + (E - v)_i - e \sin E_i$$

c. Compute $f_N(p)$:

$$\Delta t_{com} = \frac{L_2 - L_1}{n}, \quad 0 \leq L_2 - L_1 < 2\pi$$

$$f_N(p) = \Delta t_{com} - \Delta t_{obs} + P \cdot N$$

2.4 COMPUTATION OF OUTPUT QUANTITIES

The following quantities are computed, in addition to quantities already computed, for inclusion in the output once a solution is found.

a. Compute a_{xN}, a_{yN}, and ω:

$$a_{xN} = \frac{V_{z1} e \cos v_1 + U_{z1} e \sin v_1}{\sin i}$$

$$a_{yN} = \frac{U_{z1} e \cos v_1 - V_{z1} e \sin v_1}{\sin i}$$

$$\omega = \tan^{-1} \frac{a_{yN}}{a_{xN}}$$

b. Compute the time of nodal crossing, T_N, closest to, but after t_1:

$$(1 + a_{xN}) e \cos E_N = a_{xN} + e^2$$

$$(1 + a_{xN}) e \sin E_N = - \sqrt{1 - e^2} \; a_{yN}$$

$$E_N = \tan^{-1} \left[\frac{(1 + a_{xN}) e \sin E_N}{(1 + a_{xN}) e \cos E_N} \right]$$

$$e \sin E_N = \frac{-\sqrt{1 - e^2} \; a_{yN}}{1 + a_{xN}}$$

$$M_N = E_N - e \sin E_N$$

$$M_1 = L_1 - \omega - \Omega \text{ if } W_z > 0$$

$$M_1 = L_1 - \omega + \Omega \text{ if } W_z < 0$$

$$T_N = t_1 + \frac{M_N - M_1}{n}, \text{ where } 0 \leq M_N - M_1 < 2\pi$$

c. Compute, ~~q~~, $\underline{h}$, and c":

$$q = a\,(1-e)$$

$$\underline{h} = \sqrt{p}\;\underline{W}$$

$$c''\ (1/\text{min}) = \frac{-360\, n^2\, c_o}{2} \quad \text{where } c_o \text{ is in days}$$

2.5 THIRD VECTOR CHECK

If a third position fix is available, $\underline{r}_3$, it is used to check each possible solution. The position on the possible solution orbit, $\underline{r}_3'$, at the time of the 3rd position fix is computed using the Simplified General Perturbations technique*, with t_1 as epoch. The difference between the computed and the observed position is determined, $|\underline{\Delta r}| = \sqrt{(\underline{r}_3' - \underline{r}_3)\cdot(\underline{r}_3' - \underline{r}_3)}$ and compared with ϵ_4, (ϵ_4 is an input to the program). If $|\underline{\Delta r}| \leq \epsilon_4$ then the orbit is assumed to be a solution and is printed with all other solutions.

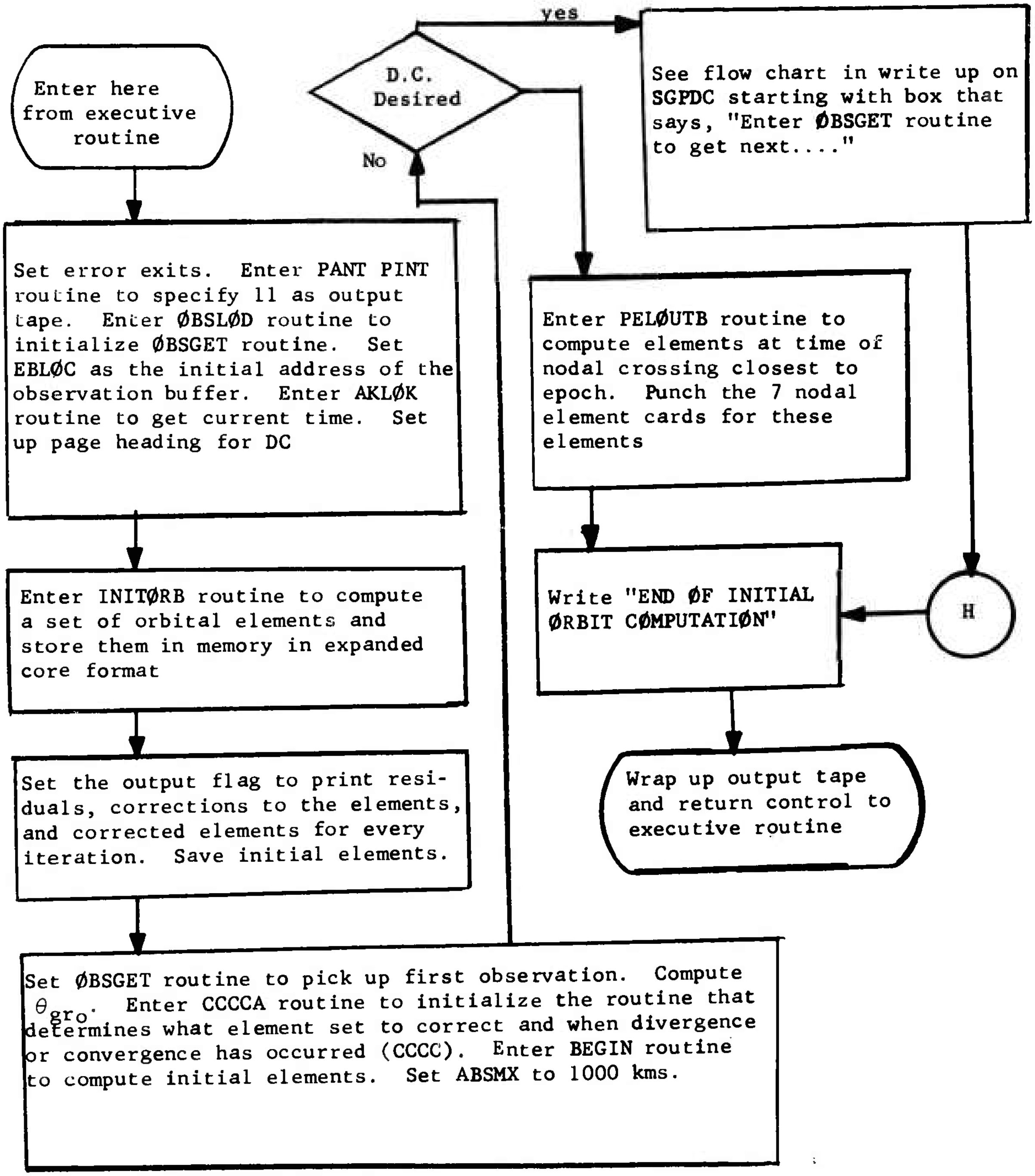
*See Section 4.49, Aeronutronic Publication U-1691, Semi-Automatic Programming System, Programming Document.

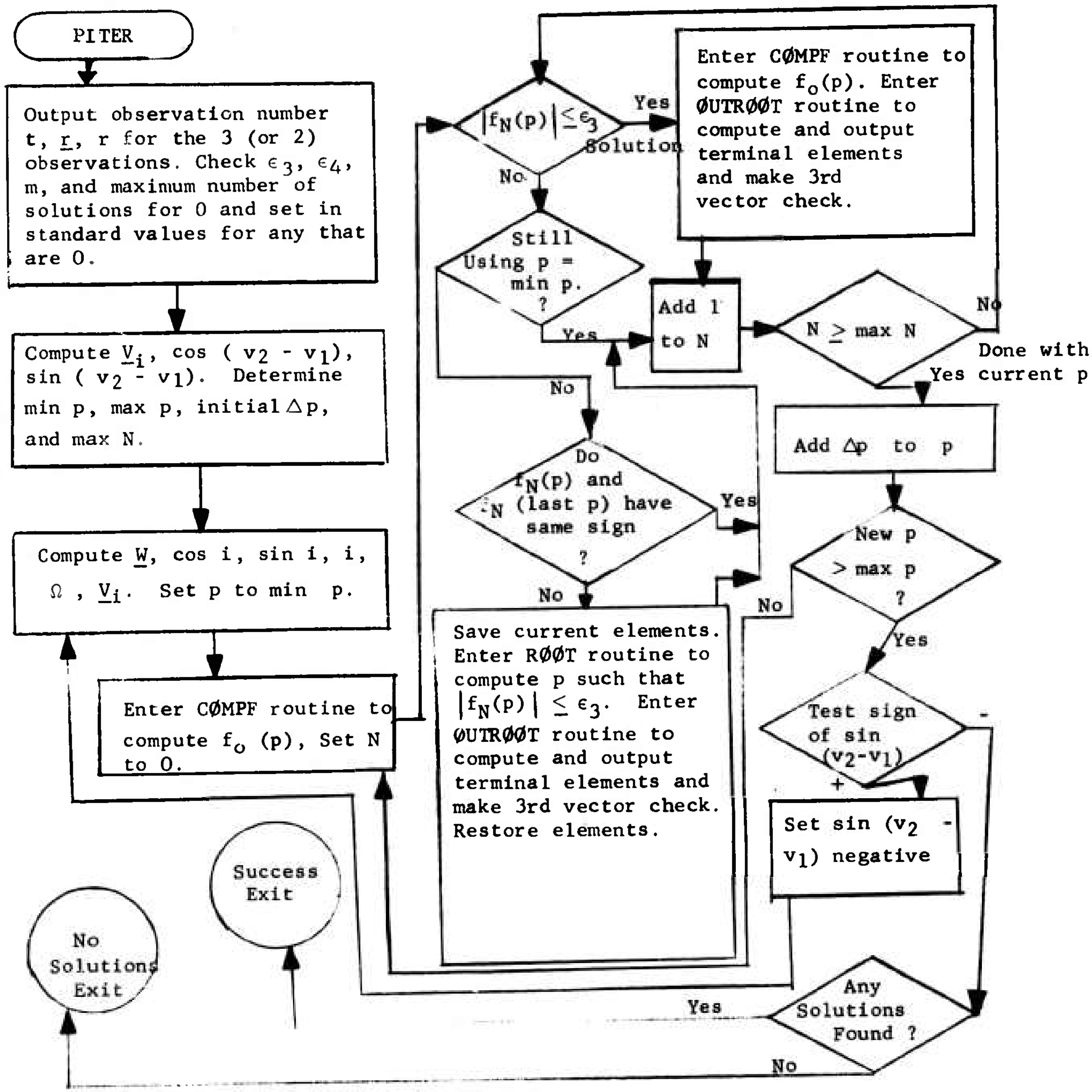
SECTION 3

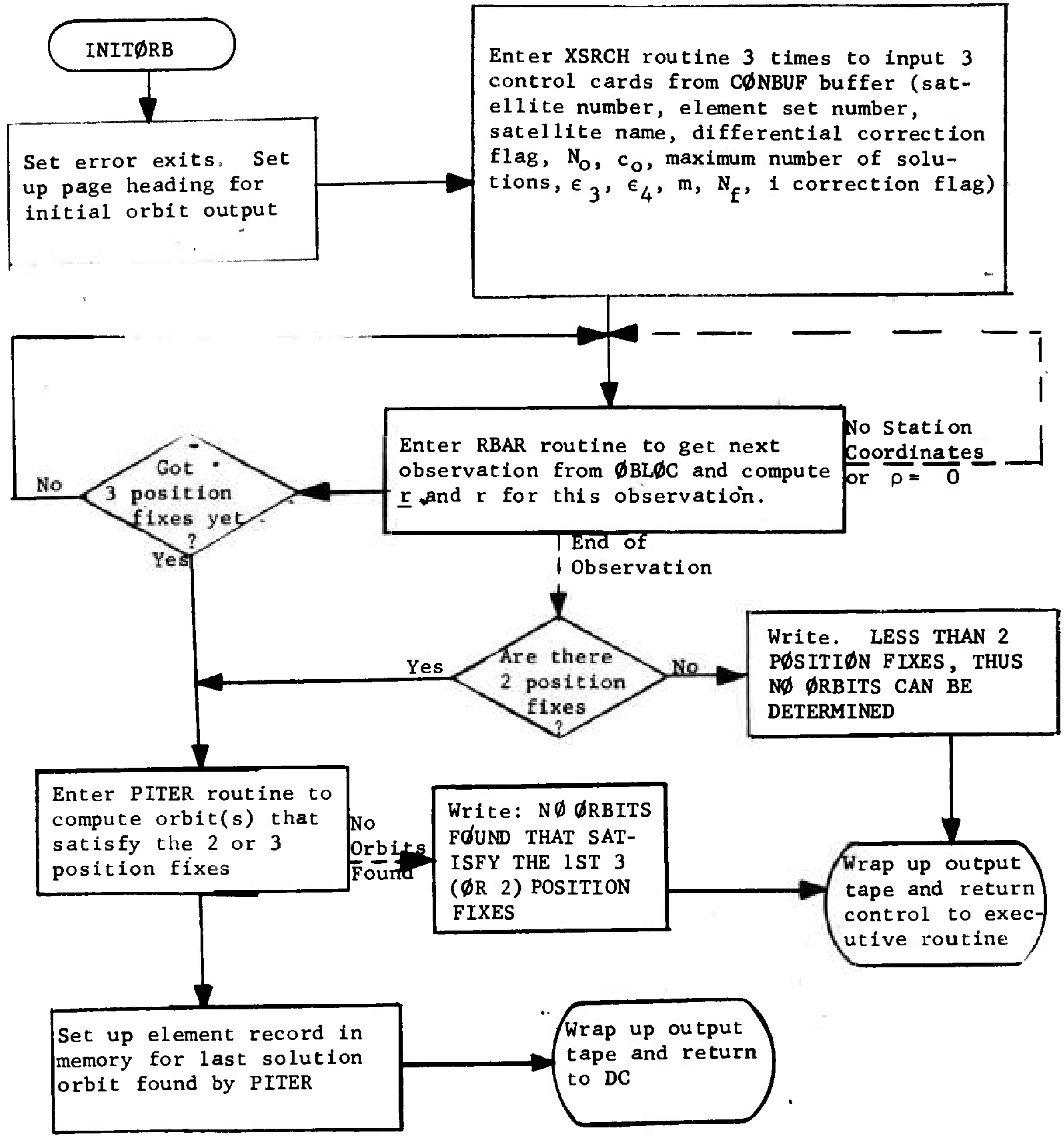
FLOW DIAGRAMS

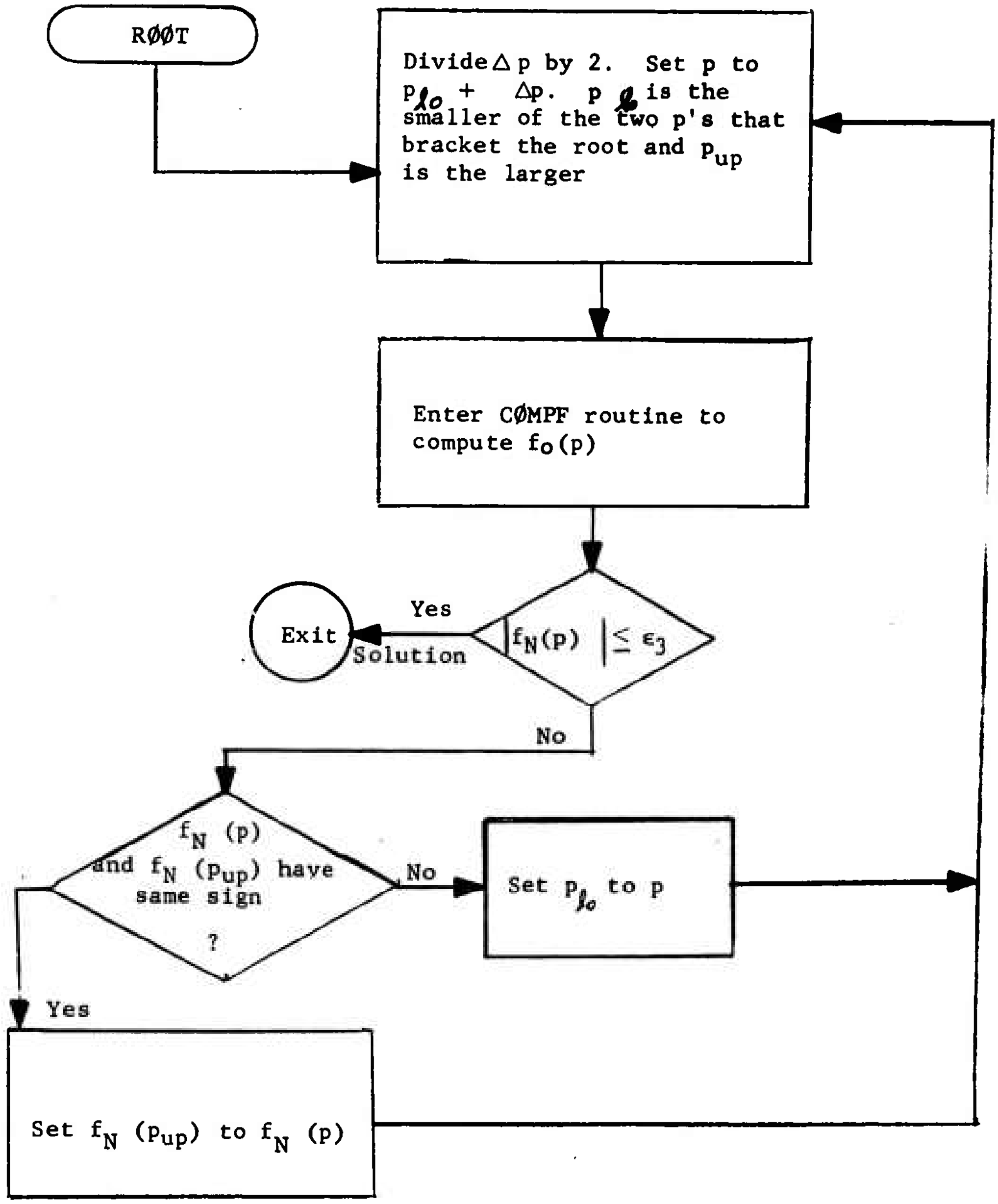
On the following pages is a flow chart of this program.

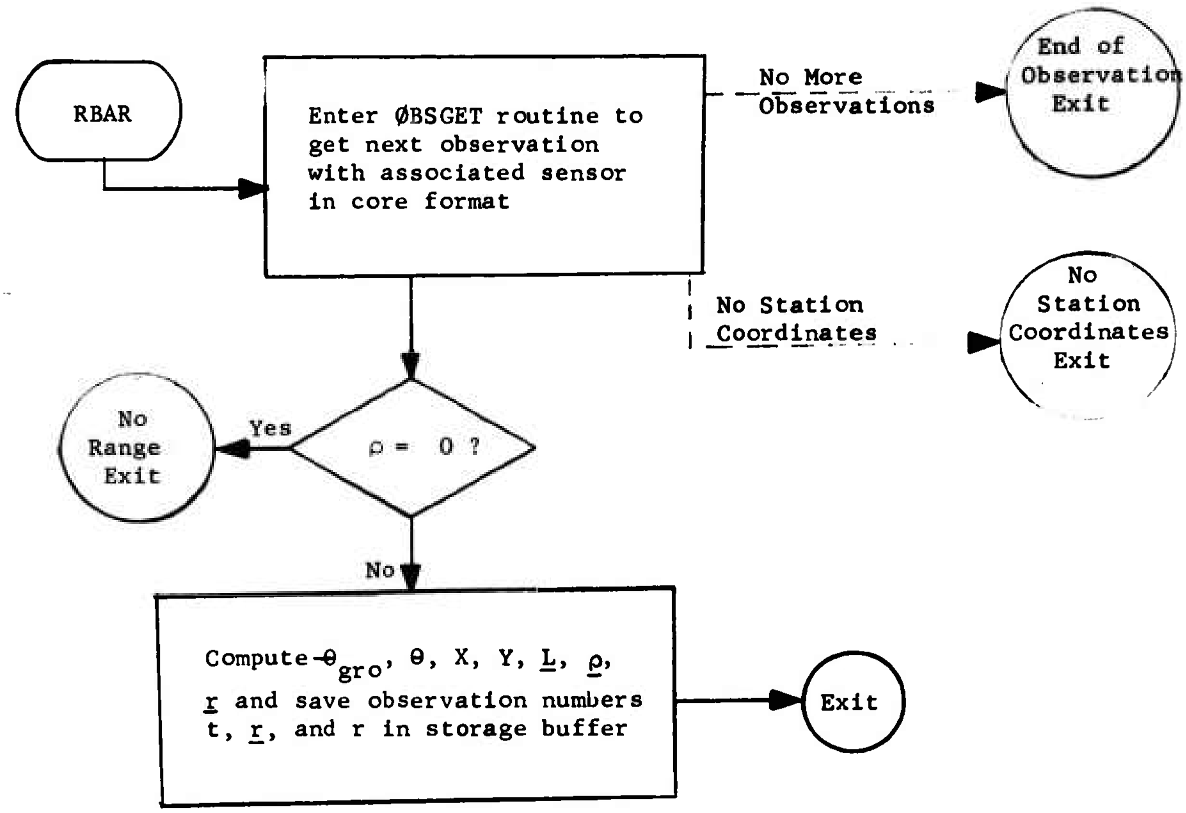
Diamonds indicate decisions. Arrows indicate direction of logical flow. If a solid arrow and one or two dotted arrows emerge from the same box, the solid arrow indicates the normal flow of logic while the dotted arrows indicate some special condition which is described next to the arrow. Ovals and circles indicate entrances and exits. The oval at the entrance to a subroutine contains within it, the symbolic name of the subroutine.

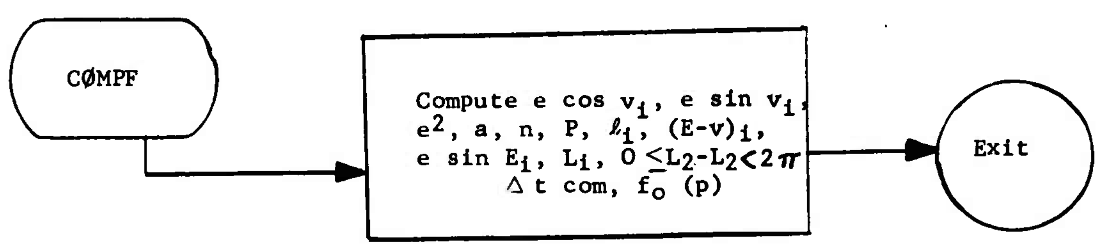

Enter here from executive routine
D.C. Desired
yes
No
See flow chart in write up on SGPDC starting with box that says, "Enter ØBSGET routine to get next...."
Set error exits. Enter PANT PINT routine to specify 11 as output tape. Enter ØBSLØD routine to initialize ØBSGET routine. Set EBLØC as the initial address of the observation buffer. Enter AKLØK routine to get current time. Set up page heading for DC
Enter PELØUTB routine to compute elements at time of nodal crossing closest to epoch. Punch the 7 nodal element cards for these elements
Enter INITØRB routine to compute a set of orbital elements and store them in memory in expanded core format
Write "END ØF INITIAL ØRBIT CØMPUTATIØN"
H
Set the output flag to print residuals, corrections to the elements, and corrected elements for every iteration. Save initial elements.
Wrap up output tape and return control to executive routine
Set ØBSGET routine to pick up first observation. Compute θ_{gro}. Enter CCCCA routine to initialize the routine that determines what element set to correct and when divergence or convergence has occurred (CCCC). Enter BEGIN routine to compute initial elements. Set ABSMX to 1000 kms.

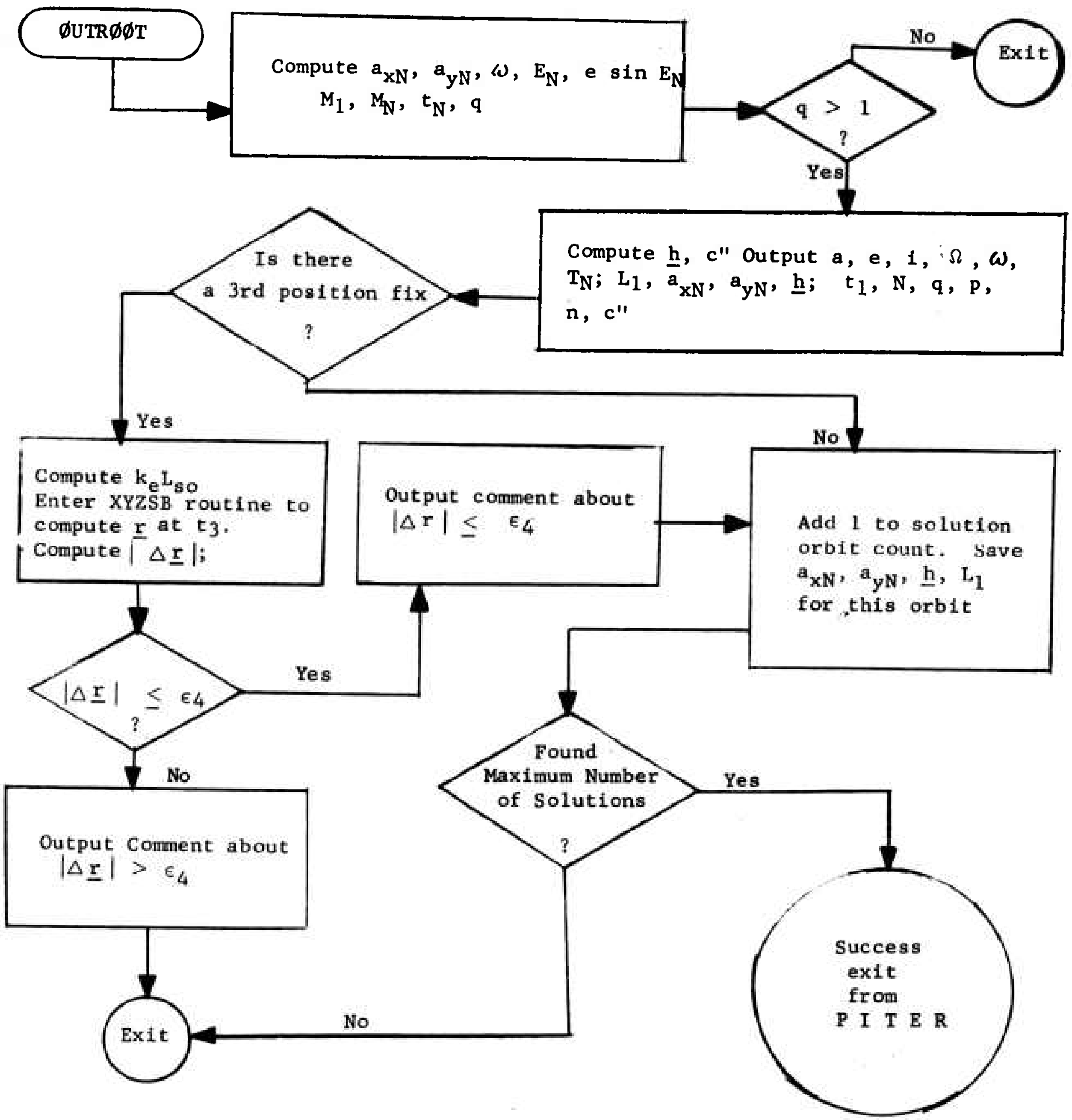

PITER
Output observation number t, r, r for the 3 (or 2) observations. Check ϵ_3, ϵ_4, m, and maximum number of solutions for 0 and set in standard values for any that are 0.
Compute $\underline{V}_i$, cos (v_2 - v_1), sin (v_2 - v_1). Determine min p, max p, initial Δp, and max N.
Compute $\underline{W}$, cos i, sin i, i, Ω , $\underline{V}_i$. Set p to min p.
Enter CØMPF routine to compute f_o (p), Set N to 0.
$|f_N(p)| \leq \epsilon_3$
Yes
Solution
No
Enter CØMPF routine to compute $f_o(p)$. Enter ØUTRØØT routine to compute and output terminal elements and make 3rd vector check.
Still Using p = min p. ?
Yes
No
Add 1 to N
N $\geq$ max N
No
Done with current p
Yes
Add Δp to p
Do $f_N(p)$ and f_N (last p) have same sign ?
Yes
No
Save current elements. Enter RØØT routine to compute p such that $|f_N(p)| \leq \epsilon_3$. Enter ØUTRØØT routine to compute and output terminal elements and make 3rd vector check. Restore elements.
New p > max p ?
No
Yes
Test sign of sin (v_2-v_1)
-
+
Set sin (v_2 - v_1) negative
Success Exit
No Solutions Exit
Any Solutions Found ?
Yes
No


INITØRB
Set error exits. Set up page heading for initial orbit output
Enter XSRCH routine 3 times to input 3 control cards from CØNBUF buffer (satellite number, element set number, satellite name, differential correction flag, N_o, c_o, maximum number of solutions, ϵ_3, ϵ_4, m, N_f, i correction flag)
Enter RBAR routine to get next observation from ØBLØC and compute r and r for this observation.
No Station Coordinates or ρ = 0
Got 3 position fixes yet?
No
Yes
End of Observation
Are there 2 position fixes?
Yes
No
Write. LESS THAN 2 PØSITIØN FIXES, THUS NØ ØRBITS CAN BE DETERMINED
Enter PITER routine to compute orbit(s) that satisfy the 2 or 3 position fixes
No Orbits Found
Write: NØ ØRBITS FØUND THAT SATISFY THE 1ST 3 (ØR 2) POSITION FIXES
Wrap up output tape and return control to executive routine
Set up element record in memory for last solution orbit found by PITER
Wrap up output tape and return to DC


RØØT
Divide Δp by 2. Set p to
p_{lo} + Δp. p_{lo} is the
smaller of the two p's that
bracket the root and p_{up}
is the larger
Enter CØMPF routine to
compute $f_o(p)$
Exit
Yes
Solution
$|f_N(p)| \leq \epsilon_3$
No
f_N (p)
and f_N (p_{up}) have
same sign
?
No
Set p_{lo} to p
Yes
Set f_N (p_{up}) to f_N (p)


RBAR
Enter ØBSGET routine to get next observation with associated sensor in core format
No More Observations
End of Observation Exit
No Station Coordinates
No Station Coordinates Exit
ρ = 0 ?
Yes
No Range Exit
No
Compute θgro, θ, X, Y, L, ρ, r and save observation numbers t, r, and r in storage buffer
Exit


CØMPF
Compute e cos vi, e sin vi, e2, a, n, P, ℓi, (E-v)i, e sin Ei, Li, 0 ≤L2-L2<2π Δ t com, fo (p)
Exit


ØUTRØØT
Compute a_{xN}, a_{yN}, ω, E_N, e sin E_N M_1, M_N, t_N, q
q > 1 ?
No
Exit
Yes
Compute $\underline{h}$, c" Output a, e, i, Ω, ω, T_N; L_1, a_{xN}, a_{yN}, $\underline{h}$; t_1, N, q, p, n, c"
Is there a 3rd position fix ?
Yes
No
Compute $k_e L_{so}$ Enter XYZSB routine to compute $\underline{r}$ at t_3. Compute $|\Delta \underline{r}|$;
Output comment about $|\Delta r| \leq \epsilon_4$
Add 1 to solution orbit count. Save a_{xN}, a_{yN}, $\underline{h}$, L_1 for this orbit
$|\Delta \underline{r}| \leq \epsilon_4$?
Yes
No
Output Comment about $|\Delta \underline{r}| > \epsilon_4$
Found Maximum Number of Solutions ?
Yes
No
Exit
Success exit from PITER

SECTION 4

OPERATING INSTRUCTIONS AND INPUT-OUTPUT

This program operates under the control of the Semi-Automatic Programming System (SPS) Executive Program using the Schedule Tape and the S File for input. A "cold start" loads the Executive Program into memory and transfers control to it. The Executive Program examines the toggle switches and determines that it should continue operating in the Schedule Tape Mode. The data cards for the program indicated on the SPSJØB card are processed, placed in the proper buffers, the program is loaded into memory, and control is transfered to the program.

For the details of SPS operation see Aeronutronic Publication U-1691.

INPUT

The makeup of the card deck that is put on the schedule tape is as follows: (Below is an example containing multiple cases)

1. JØB card*

2. Remarks card*

3. SPSJØB card

4. Three control cards for case 1

5. Any number of observation cards* for case 1 (sensor cards* also for input option 2)

*See U-1691 for format.

6. End of case card*

7. Three control cards for case 2

8. Any number of observation cards* for case 1 (sensor cards* also for input option 2)

9. End of case card*

   ⋮

10. Three control cards for last case

11. Any number of observation cards* for last case (sensor cards also for input option 2)

12. End of case card*

13. End of job card*

Card formats are as follows:

Card	Columns		
SPSJØB Card	1-8	SPSJØBΔΔ	Δ means blank
	9-16	IØRFΔΔΔΔ	
	17	0 or 2 (input option)	0 means sensors are obtained from the S file while 2 means they are obtained from sensor cards
	18	0 (output option)	
	19-72	Any identifying comments (alphanumeric)	
	73-79	Reserved for expansion	
	80	J (card type)	

* Ibid.

Card	Columns	Contents
Control Card 1	1-3	Satellite number (alphanumeric)
	4-6	Element set number
	7-8	Not used
Right	9-18	Satellite name (alphanumeric)
Adjusted	19-22	Not used
	23-36	Epoch revolution number
	37-78	Not used
	79	DC option - 0 means: do not differentially correct, $\neq$ 0 means: do differentially correct.
	80	P (card type)
Control Card 2	1-8	Not used
	9-22	c_o in days (if not known, use 0)
	23-24	M, maximum number of solutions desired
Floating Point	25-36	ϵ_3 in mins., the magnitude of the difference between the computed and the observed time between the two positions, must be less than ϵ_3.
	37-48	ϵ_4 in kms., the distance of the 3rd position fix from the computed orbit must be less than ϵ_4.
	49-60	m, number of parts to divide the interval between minimum p and maximum p into.
	61-79	Not used
	80	P (card type)
Control Card 3	1-22	Not used
	23-36	Prediction revolution number (see DC output)
	37-66	Not used
	67	i option - 0 means "correct inclination," $\neq$ 0 means "don't correct inclination"
	68-79	not used
	80	P (card type)

All columns used for identification (e.g. Control Card 1, Columns 1-3) may be left blank if the information is unknown. If M, ϵ_3, ϵ_4 and/or m are input as 0 or all blanks, they are replaced internally by 1.0, .001 mins., 100 kms., and/or 50.0 respectively. The decimal point may be punched anywhere within the floating point field.

Before transferring control to this program, the Executive Program does the following operations:

1. The 3 control cards are placed in CØNBUF in BCD character format (80 columns)

2. The observation cards are converted and placed in ØBLØC.

3. The sensor cards (input option 2) are converted and placed in SBLØC, or (input option 0) the sensors associated with the observations are read from the S file and placed in SBLØC

OUTPUT

At the top of each page the comment "INITIAL ØRBIT DETERMINATIØN, P-ITERATIØN METHØD", the date, and the page number are printed. The first comment that is printed after the page heading for each case is "THE ØBSERVATIØNS TØ BE USED ARE -" followed by a listing of observation numbers., time, x,y,z, and r for the first three (or two) observation fixes. These quantities have appropriate headings that also give the units used.

For each possible solution, i.e., each pair of p and N for which $| f_N(p) | \leq \epsilon_3$, the following quantities are printed, a,e,i, Ω, ω, T_n, L_i, a_{xN}, a_{yN}, h_x, $\bar{h}_y$, h_z, t_i, N,q,p,n, and c." Following this is a line containing the results of the third vector check. If $|\Delta \underline{r}| \leq \epsilon_4$, this line reads "MAGNITUDE ØF DELTA R BAR = .XXXXXXX $\pm$Y KMS, THEREFØRE 3RD ØBSERVATIØN DØES FIT THIS ØRBIT" If $|\Delta \underline{r}| > \epsilon_4$, the line reads "MAGNITUDE ØF DELTA R BAR = .XXXXXXX $\pm$ Y KMS, THEREFØRE 3RD ØBSERVATIØN DØES NØT FIT THIS ØRBIT". See Figure 1 for the format of the output.

There are several error conditions that cause error comments to be printed. They are described below.

1. If, during the input of the three control cards any illegal characters are encountered, "ERRØR IN INPUT CARDS" is written.

2. If, at any time during the program, exponent overflow causes a jump to location 0, "EXPØNENT ØVERFLØW FRØM LØCATIØN XXXXX" is written.

3. If, at any time during the program a subroutine error causes a jump to location 3, "SUBRØUTINE ERRØR FRØM LØCATIØN XXXXX" is written.

4. If there are less than two position fixes available, "LESS THAN 2 PØSITIØN FIXES, THUS NØ ØRBITS CAN BE DETERMINED" is written.

5. If $t_1 = t_2$, "THE TIMES ARE EQUAL THEREFØRE NØ ØRBITS CAN BE DETERMINED FROM THE 1ST TWØ PØSITIØN FIXES" followed by error note 11 is written.

6. If $t_2-t_1 \geq 2$ days, "THE TIME BETWEEN THE TWØ PØSITIØN FIXES IS NØT LESS THAN 2 DAYS, THUS THIS RØUTINE WILL ATTEMPT NØ ØRBIT DETERMINATIØN" followed by error note 11 is written.

7. If $\sin (v_2-v_1) = 0$, "THE DIFFERENCE BETWEEN THE TRUE ANØMALIES IS EQUAL TØ 0 ØR 180 DEGREES THEREFØRE NØ ØRBITS CAN BE DETERMINED FRØM THE 1ST TWØ PØSITIØN FIXES" followed by error note 11 is written.

8. If there are no values of p for which $e^2 \leq .95$, "ALL VALUES OF P GIVE E**2 > .95 THEREFØRE NØ ØRBITS CAN BE DETERMINED FRØM THE 1ST TWØ PØSITIØN FIXES" followed by error note 11 is written.

9. If max $p <$ min p, "THE MAXIMUM ALLØWABLE P IS STRICTLY LESS THAN THE MINIMUM ALLØWABLE P THEREFØRE NØ ØRBITS CAN BE DETERMINED FROM THE 1ST TWO POSITION FIXES" followed by error note 11 is written.

10. If in the subroutine RØØT,Δp becomes so small that $p + \Delta p = p$, "RØØT INDICATED BETWEEN P = X.XXXXXXXX AND P = X.XXXXXXXX FØR N = XX BUT CØULDN"T CØNVERGE. LAST P TRIED = X.XXXXXXXX, F SUB N OF P = $\pm$ .XXXXXXXX $\pm$ YY" is written.

11. If at the end of the initial orbit part of the program no solution orbits have been found, "NØ ØRBITS FØUND THAT SATISFY THE 1ST X PØSITIØN FIXES" is written.

After writing any of error notes 1 through 9 or 11 the output tape is wrapped up and control is returned to the Executive Program. After writing error note 10 the program continues normally to look for solutions.

If differential correction is not desired, the program punches the 7 standard element cards at the time of nodal crossing closest to, but after t_1, for the last solution orbit found.

If differential correction is desired then the orbit that is corrected is the very last solution found. The inclination may be corrected if desired, but the drag coefficient, c", is never corrected. During the differential correction the outputs are: all the residuals in the observations, corrections to the orbital elements, and corrected orbital elements for every iteration. The 7 standard element cards are punched for the final elements, both at the time of nodal crossing closest to t_1, and at the time of nodal crossing into the prediction revolution. For details of the above output method, reference should be made to the description of the SGPDC Program.

INITIAL ØRBIT DETERMINATIØN,P-ITERATION METHØD

THE ØBSERVATIØNS TØ BE USED ARE -

ØBS NØ	YMMDDHH MMSS.SSS	X-RADII	Y-RADII	Z-RADII	R-RADII
XXXXX	XXXXXXXX XXXX.XXX	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X
XXXXX	XXXXXXXX XXXX.XXX	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X
XXXXX	XXXXXXXX XXXX.XXX	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X

A-RADII	E	I-DEGREES	NØDE-DEGREES	ØMEGA-DEGREES	TN-DAYS SINCE 1950
±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X
L1	AXN	AYN	HX	HY	HZ
±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X
T1-DAYS SINCE 1950	REVS BETWEEN ØBS	Q-RADII	P-RADII	N-RADIANS/MIN	C DBL PRIME-1/MINS
±.XXXXXXXXXXXXX±X	XX	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X	±.XXXXXXXXXXXXX±X

MAGNITUDE ØF DELTA R BAR = .XXXXXXX±X KMS, THEREFØRE 3RD ØBSERVATIØN (DØES NØT)FIT THIS ØRBIT
or (DØES)

NØ ØRBITS FØUND THAT SATISFY THE 1ST 3 PØSITIØN FIXES

FIGURE 1 - ØUTPUT FØRMAT

SECTION 5

TEST CASES

Both cases use input option 0.

CASE 1:

INPUT:

Control Card 1	Control Card 2
Sat. no. = 163	c_o = 0.0 days
Element set no. = 012	M = 99
Sat. name = 61 SIGMA 1	ϵ_3 = 0.001 mins.
Epoch rev. no. = 1288	ϵ_4 = 100.0 kms.
DC option = 1	m = 2

Control Card 3

Prediction rev. no. = 1288

i option = 0

Observation Cards

Sta No.	Time of Obs. Year	Mo	Day	Hr	Min	Sec	h(deg) or δ(deg)	A(deg) or α(hr,min,sec)	ρ(kms)
0001	1	12	4	2	21	27.0	h = 30.5	A = 10.1	4800.0
0001	1	12	4	2	11	27.0	h = 1.7	A = 1.6	7167.0
0001	1	12	4	2	51	27.0	δ = 40.8	$\alpha = 3^h1^m36\overset{s}{.}0$	6760.0
0001	1	12	4	2	41	27.0	h = 38.0	A = 170.3	4457.0
0001	1	12	4	2	46	27.0	h = 20.2	A = 175.9	5543.0

OUTPUT:

The first outputs are the first three position fixes -

Time YMMDDHH	MMSS.SSS	x(radii)	y(radii)	z(radii)	r(radii)
61120402	2126.979	0.403031251393	0.440155155200	1.40202623326	1.52376123762
61120402	1126.981	-0.0164250853704	0.0267504054063	1.52240997878	1.52273356565
61120402	5126.993	1.09301779186	1.08379045734	-.0189203224144	1.53936598217

The only solution printed is -

$a = 1.53551604307$ radii

$e = 0.00832494892587$

$i = 91\overset{\circ}{.}1496913060$

$\Omega = 224\overset{\circ}{.}818733520$

$\omega = 89\overset{\circ}{.}7043902194$

$T_N = 4356.17523035$ days since 1950.0

$L_1 = 225\overset{\circ}{.}442972790$

$a_{xN} = 0.0000429512482776$

$a_{yN} = 0.00832483812584$

$\underline{|\Delta r|} = 25.42393$ kms.

$h_x = -0.873235460487$

$h_y = 0.878778312006$

$h_z = -0.0248623193590$

$t_1 = 4356.09128442$ days since 1950.0

$N = 0$

$q = 1.52273295039$ radii

$p = 1.53540962433$ radii

$n = 0.0390833126090$ rad/min

$c'' = 0.29 \times 10^{-10}$ 1/mins

On the following 12 pages the residuals, corrections to the orbital elements, and the corrected orbital elements for 6 iterations are printed. The first iteration corrects n only and the next 5 iterations correct n, a_{xN}, a_{yN}, U_o, Ω, and i.

The 13th page of the differential correction has the following information

Time of epoch = 338.0912842

Old rms_1 = 8.45 kms

New rms_1 = 0.98223 kms

Old rms_2 = 0 kms/sec

New rms_2 = 0 kms/sec

$\Delta n/n = 0.66858125 \times 10^{-9}$

$\Delta a_{xN} = 0.39666 \times 10^{-9}$

$\Delta a_{yN} = -0.28740 \times 10^{-9}$

$\Delta U_o = -0.31332 \times 10^{-9}$

$\Delta\Omega = 0.7159 \times 10^{-10}$

$\Delta i = 0.1180 \times 10^{-10}$

$\Delta c''$ = blank

No. of residuals used = 13

No. of residuals rejected = 2

	Old Elements (at time of nodal crossing closest to epoch	New Elements (at time of nodal crossing closest to epoch)	New Elements (at prediction rev.)
Rev. no.	1288	1288	1288
Set. no.	12	12	13
L(deg.)	136.19780	136.48960	136.48960
T_{No}(days)	338.06358	338.06354	338.06354
a(radii)	1.5355160	1.5395001	1.5395001
e	0.00832	0.01085	0.01085
i(deg)	91.149	91.151	91.151
Ω(deg)	224.817	224.813	224.813
ω(deg)	89.735	93.642	93.642
c_o(days)	0	0	0
H_q(st. miles)	2071.6	2071.8	2071.8
P_a(min)	160.819	161.445	161.445

$\dot{\Omega} = 0.044$ deg/day, $\dot{\omega} = -1.098$ deg/day

Two sets of seven standard element cards with the new elements are punched; the first set is for the time of nodal crossing closest to epoch, and the second set is for the time of nodal crossing into the prediction revolution.

The first set is as follows:

All cards have Satellite no. = 163, element set no. = 12, and card type = E

Card 1

Sat. name = 61 SIGMA 1

N_o = 1288

e = 0.0108581954

i = $91^o.151658969$

Card 2

Year of T_{No} = 1961

T_{No} = 338.0635468638 days since 1961.0

L_o = $136^o.489606287$

Card 3

P_{No} = 0.1121147138 days

Ω_o = $224^o.813728127$

ω_o = $93^o.642762117$

q_o = 1.522783910 radii

Card 4

c_o = 0.000000000029 days

$\dot{\Omega}_o$ = 0.044228236 deg/day

$\dot{\omega}_o$ = -1.098043234 deg/day

Card 5

All blank.

Card 6

a_o = 1.5395001036 radii

Standard brightness = 0.000000000

C_p = 0.0000000000

Card 7

N_f = 1288

Expiration date of Bulletin = 1960.0 (meaningless)

ISTØP = 0

The second set is the same as the first set except that the expiration date of the Bulletin is December 4, 1961, $1^h\ 31^m\ 30^s.45$ and the element set no. = 13.

CASE 2

INPUT:

Control Card 1

Sat. No. = 034

Element set no. = 099

Sat. name = 60 EPSI 1

Epoch rev. no. = 8670

DC option = 0

Control Card 2

$c_o = -0.11251 \times 10^{-6}$

M = 99

ϵ_3 = .001 mins

ϵ_4 = 100 kms.

m = 2

Control Card 3

Prediction rev. no. = 8670

i option = 0

Observation Cards

Sta. No.	Time of Obs Year	Mo.	Day	Hr.	Min.	Sec.	h(deg)	A(deg)	ρ(kms)
0316	1	11	25	4	10	26.0	5.8	328.75	1834.67
0316	1	11	25	5	45	6.0	5.8	318.75	1760.54
0316	1	11	26	3	05	24.0	1.6	321.25	2149.71

OUTPUT:

The three position fixes:

Time YMMDDHH	MMSS.SSS	x(radii)	y(radii)	z(radii)	r(radii)
61112504	1025.980	0.0908385425456	-0.445447206264	0.963478327438	1.06534752121
61112505	4505.987	0.267520165478	-0.415019701875	0.939419196686	1.06128074461
61112603	0523.986	-0.0972024607181	-0.444682810921	0.958790981211	1.06135350652

Two possible solutions are printed, the first is the true solution.

First Solution -

$a = 1.05957282532$ radii	$h_x = -0.0998981992772$
$e = 0.0227595478820$	$h_y = 0.926056950702$
$i = 64^{\circ}.8369177419$	$h_z = 0.437564682332$
$\Omega = 186^{\circ}.156960786$	$t_1 = 4347.17391151$ days since 1950.0
$\omega = 197^{\circ}.405908140$	$N = 1$
$T_N = 4347.22091013$ days since 1950.0	$q = 1.03545742691$ radii
$L_1 = 280^{\circ}.974228400$	$p = 1.05902396957$ radii
$a_{xN} = -0.0217173765006$	$n = 0.0681830911664$ rad/min.
$a_{yN} = -0.00680827280506$	$c'' = 0.190794808906 \times 10^{-7}$ 1/mins.
$\lvert \underline{\Delta r} \rvert = 2506.851$ kms.	

Second Solution -

$a = 1.09746945963$ radii	$h_x = 0.101620034315$
$e = 0.0384727603814$	$h_y = -0.942018373054$
$i = 115^{\circ}.163082256$	$h_z = -0.445106502215$
$\Omega = 6^{\circ}.15696106455$	$t_1 = 4347.17391151$ days since 1950.0
$\omega = 45^{\circ}.7917417283$	$N = 0$
$T_N = 4347.26605452$ days since 1950.0	$q = 1.05524678016$ radii
$L_1 = 78^{\circ}.6716120957$	$p = 1.09584503690$ radii
$a_{xN} = 0.0268258411204$	$n = 0.0646821189089$ rad/min.
$a_{yN} = 0.0275776638241$	$c'' = 0.171704492357 \times 10^{-7}$ 1/mins
$\lvert \underline{\Delta r} \rvert = 5181.064$ kms	

No set of 7 standard element cards should be punched because $\lvert \underline{\Delta r} \rvert >$ 100 kms. for both possible solutions.

SECTION 6

EXPERIMENTATION

Tables I through VII contain the results of several cases. The results also include a direct comparison with the SPADATS elements. Tables II, VI, and VII use simulated observations and therefore give an excellent indication of the true accuracy of the program. The experimentation contains cases with a range of eccentricities from 0.005 to 0.2 and with up to 13 revolutions. In all cases, the number of revolutions, N, and the time, Δt, between the two observations is indicated. $\epsilon_3 = 0.001$ mins. for all cases.

In general, for each possible number of revolutions between the first two position fixes, there is both a direct and retrograde two body orbit that passes between the two positions in the proper duration of time. The third vector check is useful for isolating the true solution orbit. Comparing each possible residual, $|\Delta \underline{r}|$, with a fixed ϵ_4 is not the most efficient way to isolate the true solution from the spurious ones. It is nearly impossible to assign a value to ϵ_4 that will be larger than the $|\Delta \underline{r}|$ for the true solution and yet smaller than the $|\Delta \underline{r}|$'s for all the other possible solutions. For instance, if the third observation is from the same part of the orbit as the first two (and this is quite likely if the third observation is from the same station) the $|\Delta \underline{r}|$'s for many of the possible solutions can be as small, or smaller, than the $|\Delta \underline{r}|$ for the true solution. Similarly, if the third observation is far removed from the first two in time, the $|\Delta \underline{r}|$ can be quite large for even the true solution. In the bottom two cases of Table II, some of the spurious solutions yielded a smaller $|\Delta \underline{r}|$ than did the true solution.

A better way to isolate the true solution is to pick the orbit with the smallest $|\Delta \underline{r}|$. This does not always yield the true solution, as witness the aforementioned two cases. However, in all the other cases

performed in the experimentation, the smallest $|\Delta \underline{r}|$ was associated with the correct solution. Picking the orbit with the smallest $|\Delta \underline{r}|$ is reliable if the three observations are chosen judiciously. The first two positions should be picked from the same revolution, if possible, and if not, as close together as feasible. The third observation should be from a different station than the first two in order for it to be on a different part of the orbit.

An examination of the cases performed in the experimentation indicates that some cases show a rather large deviation from the SPADATS data for the two parameters e and ω. This is due to the fact that there is a singularity in the method for e = 0. Therefore, if the orbit is close to being circular, there is a liklihood of errors being introduced into the results. These errors may be aggravated by two parameters. If $\sin(v_2 - v_1)$ is very close to zero (i.e., $v_2 - v_1 < 4^o$, and/or p/r_1 is close to unity the errors will be magnified. For example, in Table II for "60 Nu 2, Δt - 1.470 mins," the value for $\sin(v_2 - v_1)$ is quite small and the error in ω is 37^o. On the other hand, for the same satellite and $\Delta t = 3^h\ 52^m .904$, $\sin(v_2 - v_1)$ is quite large and the error in ω is 1^o. Obviously, there is nothing that can be done about p/r_1 to improve the accuracy, but $\sin(v_2 - v_1)$ can be improved in some cases. If there is a choice in the selection of the observations, the two that give as different positions on the orbit as possible should be chosen. They should both be on the same revolution, if possible, in order to be consistent with the suggestions concerning the magnitude of $|\Delta \underline{r}|$. If this is done, and p/r_1 does not turn out to be very close to unity, the program will handle successfully orbits with very low eccentricities.

The smallest error, for any of the cases shown here, in the semimajor axis, a, or perigee distance, q, is .0004 radii. To achieve this same accuracy in p, the experimentation indicates that ϵ_3 could be as large as 0.5 mins. This would save approximately 10 iterations as compared with using ϵ_3 = 0.001 mins.

TABLE I

60 Epsilon 1, N = 1, Δt = 94.7 mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.0404	1.0355	0.0049
e	0.01546	0.02276	0.00730
i(degrees)	65.018	64.837	0.181
Ω(degrees)	186.424	186.157	0.267
ω (degrees)	211.092	197.406	13.686
T_n(days since 1961.0)	329.22115	329.22091	0.00024 (0.34 mins)

60 Epsilon 1, N = 3, Δt = 284.1 mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.0404	1.0440	0.0036
e	0.01546	0.01252	0.00294
i(degrees)	65.018	65.114	0.096
Ω(degrees)	186.645	186.032	0.613
ω (degrees)	211.120	195.002	16.118
T_n(days since 1961.0)	329.15741	329.15744	0.00003 (0.04 mins)

60 Eta 2, N = 6, Δt = 709.5 mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.0961	1.0994	0.0033
e	0.03076	0.02807	0.00269
i(degrees)	66.770	66.687	0.083
Ω(degrees)	217.514	216.714	0.800
ω (degrees)	220.194	228.182	7.988
T_n(days since 1961.0)	330.15650	330.15654	0.0004 (0.06 mins)

61 Sigma 1, N = 10, Δt = 1739.8 mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.5259	1.5267	0.0008
e	0.00923	0.00898	0.00025
i(degrees)	91.131	91.155	0.024
Ω(degrees)	223.586	223.646	0.060
ω (degrees)	106.290	116.180	9.890
T_n(days since 1961.0)	315.17846	315.17836	0.00010 (0.14 mins)

TABLE II 61 SIGMA 1

$N = 0$, $\Delta t = 10.0$ mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.5398	1.5355	0.0043
e	0.01069	0.00832	0.00237
i(degrees)	91.198	91.150	0.048
Ω(degrees)	224.778	224.819	0.041
ω (degrees)	93.384	89.704	3.680
T_n(days since 1961.0)	338.17589	338.17523	0.00046 (0.67 mins)

$N = 1$, $\Delta t = 165.0$ mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.5398	1.5404	0.0006
e	0.01069	0.01152	0.00083
i(degrees)	91.198	91.149	0.049
Ω(degrees)	224.778	224.979	0.201
ω (degrees)	93.384	85.428	7.956
T_n(days since 1961.0)	338.17589	338.17577	0.00012 (0.17 mins)

$N = 2$, $\Delta t = 325.0$ mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.5398	1.5404	0.0006
e	0.01069	0.01152	0.00083
i(degrees)	91.198	91.148	0.050
Ω(degrees)	224 778	225.232	0.454
ω (degrees)	93.384	85.091	8.293
T_n(days since 1961.0)	338.17589	338.17577	0.00012 (0.17 mins)

$N = 4$, $\Delta t = 645.0$ mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.5398	1.5404	0.0006
e	0.01069	0.01180	0.00111
i(degrees)	91.198	91.156	0.042
Ω(degrees)	224.778	223.458	0.320
ω (degrees)	93.384	103.821	10.437
T_n(days since 1961.0)	338.17589	338.17565	0.00024 (0.35 mins)

TABLE II 61 SIGMA II
(Continued)

$N = 6, \Delta t = 1090.0$ mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.5398	1.5404	0.0006
e	0.01069	0.01191	0.00122
i(degrees)	91.198	91.246	0.048
Ω (degrees)	224.778	224.826	0.048
ω (degrees)	93.384	95.972	2.588
T_n (days since 1961.0)	338.17589	338.17576	0.00013 (0.19 mins)

$N = 8, \Delta t = 1295.0$ mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.5398	1.5405	0.007
e	0.01069	0.01167	0.00098
i(degrees)	91.198	91.146	0.052
Ω(degrees)	224.778	225.487	0.709
ω (degrees)	93.384	100.110	6.726
T_n (days since 1961.0)	338.17589	338.17567	0.00022 (0.32 mins)

TABLE III

60 Nu 2, N = 0, Δt = 1.470 mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.1674	1.1799	0.0125
e	0.01894	0.01760	0.00134
i(degrees)	28.22	28.22	0.00
Ω (degrees)	284.883	284.068	0.185
ω (degrees)	281.228	318.774	37.546
T_n (days since 1962.0)	5.99040	5.99182	0.00142 (2.04 mins)

60 Nu 2, N = 2, $\Delta t = 3^h\ 52.^m904$

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.1674	1.1664	0.0010
e	0.01894	0.02123	0.00229
i(degrees)	28.22	28.14	0.08
Ω (degrees)	283.891	283.542	0.349
ω (degrees)	281.215	282.417	1.202
T_n (days since 1962.0)	5.99040	5.99035	0.00005 (0.07 mins)

60 Xi 1, N = 0, Δt = 1.133 mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.2104	1.2132	0.0028
e	0.12000	0.11816	0.00184
i(degrees)	49.92	49.89	0.03
Ω (degrees)	301.317	301.603	0.286
ω (degrees)	231.657	226.343	2.314
T_n (days since 1962.0)	26.14511	25.14532	0.00021 (0.31 mins)

60 Xi 1, N = 0, Δt = 1.231 mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.2104	1.1970	0.0134
e	0.12000	0.13250	0.01250
i(degrees)	49.92	50.21	0.29
Ω (degrees)	301.587	301.038	0.549
ω (degrees)	231.432	234.458	3.026
T_n (days since 1962.0)	26.06703	26.06543	0.00160 (2.30 mins)

TABLE IV

60 Pi 1, N = 0, Δt = 1.122 mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.1058	1.1060	0.002
e	0.00946	0.00960	0.00014
i(degrees)	48.57	48.69	0.12
Ω (degrees)	8.344	7.911	0.433
ω (degrees)	250.369	236.382	13.987
T_n(days since 1962.0)	10.24803	10.24793	0.00010 (0.15 mins)

60 Pi 1, N = 13, $\Delta t = 22^h 18^m.876$

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.1058	1.0876	0.0182
e	0.00947	0.02407	0.01460
i(degrees)	48.57	51.04	2.47
Ω (degrees)	327.264	22.324	55.060
ω (degrees)	287.285	223.865	63.420
T_n(days since 1962.0)	19.11103	19.11051	0.00052 (0.74 mins)

60 Iota 1, N = 0, Δt = 1.296 mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.2373	1.2244	0.0129
e	0.05751	0.05900	0.00149
i(degrees)	47.28	46.74	0.54
Ω (degrees)	356.493	355.233	1.260
ω (degrees)	271.611	264.449	7.162
T_n(days since 1962.0)	30.08049	30.07904	0.00145 (2.09 mins)

60 Iota 1, N = 0, Δt = 0.813 mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.2373	1.2630	0.0257
e	0.05751	0.02969	0.02782
i(degrees)	47.28	47.86	0.58
Ω (degrees)	355.959	354.653	1.306
ω (degrees)	272.123	290.813	18.690
T_n(days since 1962.0)	30.24188	30.24456	0.00268 (3.86 mins)

TABLE V

60 Iota 2, N = 0, Δt = 0.888 mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.2499	1.2579	0.0080
e	0.01176	0.02182	0.01006
i(degrees)	47.22	47.34	0.12
Ω (degrees)	35.263	35.647	0.384
ω (degrees)	167.909	319.155	151.246
T_n(days since 1962.0)	30.12975	30.13093	0.00118 (1.70 mins)

60 Iota 5, N = 0, Δt = 0.910 mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.2523	1.2891	0.0368
e	0.01212	0.03581	0.02369
i(degrees)	47.20	47.77	0.57
Ω (degrees)	47.520	48.682	1.162
ω (degrees)	159.549	88.026	71.523
T_n(days since 1962.0)	30.13276	30.13657	0.00381 (5.48 mins)

61 Alpha 1, N = 0, Δt = 1.520 mins

Element	SPADATS	p-Iteration	Magnitude of Difference
a(radii)	1.0800	1.0900	0.0100
e	0.00518	0.01527	0.01007
i(degrees)	97.40	97.10	0.30
Ω (degrees)	113.970	113.683	0.287
ω (degrees)	58.058	59.571	1.539
T_n(days since 1962.0)	16.21038	16.21128	0.00090 (1.29 mins)

TABLE VI SIMULATED OBSERVATIONS

$N = 0, \Delta t = 2.0$ mins

Element	True Elements	p-Iteration	Magnitude of Difference
a(radii)	1.5	1.5046	0.0046
e	0.2	0.19717	0.00283
i(degrees)	30.0	30.189	0.189
Ω (degrees)	29.352	28.895	0.457
ω (degrees)	31.028	32.548	1.520
T_n(days since 1962.0)		66.43099	

$N = 0, \Delta t = 4.0$ mins

Element	True Elements	p-Iteration	Magnitude of Difference
a(radii)	1.5	1.5011	0.0011
e	0.2	0.19921	0.00079
i(degrees)	30.0	29.997	0.003
Ω (degrees)	29.352	29.193	0.159
ω (degrees)	31.028	31.606	0.578
T_n(days since 1962.0)		66.43081	

$N = 0, \Delta t = 10.0$ mins

Element	True Elements	p-Iteration	Magnitude of Difference
a(radii)	1.5	1.4996	0.0004
e	0.2	0.20010	0.00010
i(degrees)	30.0	30.023	0.023
Ω (degrees)	29.353	29.153	0.200
ω (degrees)	31.028	31.334	0.306
T_n(days since 1962.0)		66.43069	

$N = 0, \Delta t = 20.0$ mins

Element	True Elements	p-Iteration	Magnitude of Difference
a(radii)	1.5	1.5011	0.0011
e	0.2	0.19946	0.00054
i(degrees)	30.0	29.981	0.019
Ω (degrees)	29.352	29.218	0.134
ω (degrees)	31.028	31.570	0.542
T_n(days since 1962.0)		66.43082	

$N = 0, \Delta t = 36.0$ mins

Element	True Elements	p-Iteration	Magnitude of Difference
a(radii)	1.5	1.5011	0.0011
e	0.2	0.19936	0.00064
i(degrees)	30.0	29.968	0.032
Ω (degrees)	29.352	29.225	0.127
ω (degrees)	31.028	31.738	0.710
T_n(days since 1962.0)		66.43084	

TABLE VII SIMULATED OBSERVATIONS

N = 1, Δt = 22.0 mins

Element	True Elements	p-Iteration	Magnitude of Difference
a(radii)	1.5	1.4992	0.0008
e	0.2	0.19854	0.00146
i(degrees)	30.0	29.997	0.003
Ω (degrees)	29.676	29.785	0.109
ω (degrees)	30.514	31.169	0.655
T_n(days since 1962.0)		66.21533	

N = 2, Δt = 32.0 min

Element	True Elements	p-Iteration	Magnitude of Difference
a(radii)	1.5	1.4991	0.0009
e	0.2	0.19916	0.00084
i(degrees)	30.0	29.919	0.081
Ω (degrees)	29.676	29.429	0.247
ω (degrees)	30.514	31.036	0.522
T_n(days since 1962.0)		66.21527	

N = 4, Δt = 15.0 mins

Element	True Elements	p-Iteration	Magnitude of Difference
a(radii)	1.5	1.4991	0.0009
e	0.2	0.19988	0.00012
i(degrees)	30.0	29.217	0.783
Ω (degrees)	29.838	29.031	0.807
ω (degrees)	30.257	31.573	1.316
T_n(days since 1962.0)		66.10752	

N = 10, Δt = 1.0 mins

Element	True Elements	p-Iteration	Magnitude of Difference
a(radii)	1.5	1.4992	0.0008
e	0.2	0.19990	0.00010
i(degrees)	30.0	31.097	1.097
Ω(degrees)	29.838	31.313	1.475
ω (degrees)	30.258	30.435	0.177
T_n(days since 1962.0)		66.10792	

SECTION 7

PROGRAM STATUS

7.1 LIMITATIONS

a. Inclination must not be equal to zero.

b. The orbit must have an eccentricity greater than zero but equal or less than $\sqrt{.95}$.

c. The time of the two observations, which determine the orbit, must not be equal to zero.

d. The difference between the true anomalies of the two observations must not be equal to zero or even multiples of π .

e. The time between the two observations must be less than two days.

7.2 IMPROVEMENTS UNDER CONSIDERATION

a. As was indicated in Section 6, the correct selection of the three observations to use for this orbit determination scheme is important for correct solutions. There is no reason why this selection cannot be done inside the program rather than leaving it up to an operator or analyst.

b. Taking the solution with the minimum $|\Delta \underline{r}|$ as the true solution would be a better way of isolating the true solution than comparing the various $|\Delta \underline{r}|$'s with a fixed ϵ_4.

c. Changing the equation for P_2max to read:

$$P_2max \leq 1.975 \times (\text{the smaller of } r_1 \text{ or } r_2)$$

would conform with the second limitation mentioned above.

d. Max N can be constantly updated to conform with the value of the previously selected p. This will reduce the number of spurious solutions.

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Unclassified Report

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1. Artificial Earth Satellites
2. Astrodynamics
3. Computer Programming

I. 496L SPO.
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